Code No: 133BD **R16**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, May/June - 2019

MATHEMATICS – IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT MCT, ETM MMT, AE, MIE, PTM, CEE, MSNT)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) State the necessary and sufficient conditions for a function $f(z) = u + iv$ to be analytic. [2]

b) Show that $f(z) = |z|^2$ is not analytic at any point. [3]

c) State Cauchy's integral theorem. [2]

d) Find the poles and the residues at the poles of the function $f(z) = \frac{e^z}{\cos \pi z}$. [3]

e) Define bilinear transformation and cross ratio. [2]

f) Find the image of the circle $|z| = 2$, under the transformation $w = z + 3 + 2i$. [3]

g) State Fourier integral theorem. [2]

h) Expand $f(x) = \pi x - x^2$ in a half range sine series in $(0, \pi)$. [3]

i) Classify the partial differential equation $u_{xx} + 6u_{xy} + 2u_{yy} + 2u_x - 2u_y + u = x^2 y$. [2]

j) Write the three possible solutions of the heat equation.
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$$ [3]

PART-B

(50 Marks)

2.a) If $f(z)$ is a regular function of z, prove that
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2.$$

b) Let $f(z) = u(r, \theta) + iv(r, \theta)$ be an analytic function. If $u = -r^3 \sin 3\theta$, then construct the corresponding analytic function $f(z)$ in terms of z. [5+5]

OR

3.a) Show that the function $f(z)$ defined by
$$f(z) = \begin{cases} \frac{x^2 y^3 (x+iy)}{x^6 + y^{10}} & for\ z \neq 0 \\ f(0) = 0 \end{cases}$$, is not analytic at the origin, even though it satisfies the Cauchy-Riemann equations at the origin.

b) Determine the analytic function whose real part is $\log \sqrt{x^2 + y^2}$. [5+5]

4. Represent the function $\frac{1}{z^2-4z+3}$ in the domain
(a) $1<|z|<3$ (b) $|z|<1$. [10]

OR

5.a) Expand the function $f(z)=\frac{z}{(z+1)(z+2)}$ about $z=-2$, and name the series thus obtained.

b) Evaluate $\oint_C \frac{e^z}{(z+3)(z+2)}dz$, where C is the circle $|z-1|=\frac{1}{2}$. [5+5]

6. Evaluate the integral using contour integration $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$. [10]

OR

7. Show that the transformation $w=i\frac{1-z}{1+z}$ transforms the circle $|z|=1$ into the real axis of w plane and the interior of the circle $|z|<1$ into the upper half of the w plane. [10]

8. Find the Fourier transform of $f(x)=\begin{cases}1-x^2, & if\ |x|<1\\ 0, & if\ |x|>1\end{cases}$. Hence evaluate
$\int_0^{\infty}\frac{x\cos x-\sin x}{x^3}\cos\frac{x}{2}dx$. [10]

OR

9.a) Obtain the half range cosine series for
$f(x)=\begin{cases}kx, & for\ 0\le x<\frac{L}{2}\\ k(L-x), & for\ \frac{L}{2}\le x\le L\end{cases}$.

b) Find the Fourier sine transform of $f(x)=e^{-|x|}$. Hence show that $\int_0^{\infty}\frac{x\sin mx}{x^2+1}dx=\frac{\pi}{2}e^{-m}$.
[5+5]

10. A string is stretched and fastened to two points L apart. Motion is started by displacing the string in the form $y=a\sin\frac{\pi x}{L}$ from which it is released at time $t=0$. Find the displacement of any point at a distance x from one end at time t. [10]

OR

11. Write down the one dimensional heat equation. Find the temperature $u(x,t)$ in a slab whose ends $x=0$ and $x=L$ are kept at zero temperature and whose initial temperature $f(x)$ is given by
$f(x)=\begin{cases}k, & when\ 0<x<\frac{1}{2}L\\ 0, & when\ \frac{1}{2}L<x<L\end{cases}$. [10]

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